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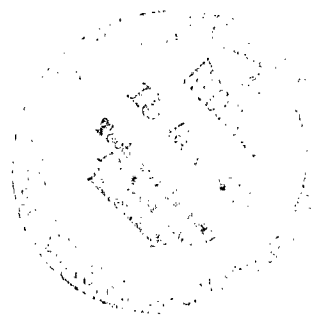
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PROPOSED TECHNIQUE FOR LAUNCHING INSTRUMENTED BALLOONS INTO TORNADOES

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1. Report No. NASA TN D-6503	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle PROPOSED TECHNIQUE FOR LAUNCHING INSTRUMENTED BALLOONS INTO TORNADOES		5. Report Date November 1971
		6. Performing Organization Code
7. Author(s) Frederick C. Grant		8. Performing Organization Report No. L-7933
9. Performing Organization Name and Address NASA Langley Research Center Hampton, Va. 23365		10. Work Unit No. 188-36-56-02
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		13. Type of Report and Period Covered Technical Note
		14. Sponsoring Agency Code
15. Supplementary Notes		
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17. Key Words (Suggested by Author(s)) Balloons Tornadoes	18. Distribution Statement Unclassified - Unlimited	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 21
		22. Price* \$3.00

PROPOSED TECHNIQUE FOR LAUNCHING INSTRUMENTED BALLOONS INTO TORNADOES

By Frederick C. Grant
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SUMMARY

A method is proposed to introduce instrumented balloons into tornadoes by means of the radial pressure gradient, which supplies a buoyancy force driving to the center. Presented are analytical expressions, verified by computer calculations, which show the possibility of introducing instrumented balloons into tornadoes at or below the cloud base. The times required to reach the center are small enough that a large fraction of tornadoes are suitable for the technique. An experimental procedure is outlined in which a research airplane puts an instrumented, self-inflating balloon on the track ahead of the tornado. The uninflated balloon waits until the tornado closes to, typically, 750 meters; then it quickly inflates and spirals up and into the core, taking roughly 3 minutes. Since the drive to the center is automatically produced by the radial pressure gradient, a proper launch radius is the only guidance requirement. At no time need the research airplane fly closer than several kilometers to the core.

INTRODUCTION

Direct measurements of wind speed, pressure, temperature, and other parameters inside a tornado are impractical when ground-based instrumentation is used. This is hardly surprising because of the rarity of tornadoes, the destructiveness of tornado winds, and the narrow area swept by them. In short, the funnel must pass directly over the instruments, which must survive the passage. The possibility of introducing instrumented balloons into the cores of tornadoes is shown in this paper.

Instead of waiting for the tornado to come to the instruments, the instruments may be carried to the tornado and, as far as possible, shielded from the winds. A combination of old and newer technology makes this feasible. An airplane can be used to drop a self-inflating balloon with instrumentation on the track ahead of a tornado. If a drop is made with sufficient accuracy, the tornado will enter a target area centered on the balloon, which will inflate and rise.

Because of the radial pressure gradient, the balloon will spiral quickly into the core of the tornado while telemetering data. Radar tracking will yield the balloon trajectory. The diameter of the target area may be several hundred meters, whereas the nearest approach of the airplane may be several kilometers. Dropping of several payloads at target-diameter intervals across the track would insure capture of at least one balloon if the drop were inaccurate or, alternatively, would allow the distance of closest approach of the airplane to be increased in rough proportion to the number of payloads dropped.

Instrumented constant-volume balloons have been used for probing thunderstorms for some time. (See refs. 1 and 2.) The transponders have been light (roughly 100 grams), and the envelopes have been made of Mylar. (A more rugged envelope will be needed for launches into tornado vicinities.) Occasionally, these balloons have entered and ascended the large-scale (diameter of kilometers) core of the storm (ref. 3), but direct insertion into the funnel of a tornado has not been attempted. A tornado probe consisting of a sequence of instrumented 2.75-inch (7-cm) aircraft rockets was suggested in reference 4.

Calculations of the target-area size, time to the center, and altitude at the center are presented in this paper. The tornado model is very simple: a vertical line vortex in an atmosphere with exponential density variation. The results should be conservative in that existence of inward flows will decrease the time to the center and increase the target area. The aim is to obtain typical values of the basic variables of the problem and to define the range of dynamical feasibility.

SYMBOLS

A	cross-sectional area of balloon, $\pi D^2/4$, m^2
C_D	aerodynamic drag coefficient based on cross-sectional area
C_D'	aerodynamic drag coefficient of n balloons of diameter d based on cross-sectional area
D	diameter of balloon, m
d	diameter of n small balloons with total volume, $\pi D^3/6$, m
F_D	drag force, N

F_r, F_z, F_θ	force applied in radial, vertical, and tangential directions, respectively, N
g	acceleration due to gravity, 9.80665 m/s ²
L	Lagrangian function, $T - V$, N-m
M	inertial mass of balloon, kg
M'	virtual mass, kg
n	number of balloons of diameter d
$\frac{\partial p}{\partial r}, \frac{\partial p}{\partial z}$	radial and vertical pressure gradients, respectively, N/m ³
Q_i	generalized force
q_i	generalized coordinate
r	radial position from vortex, m
r_0	launch radius of balloon, m
r_t	transition radius, m
S	lead distance of center of balloon launch circle, m
T	kinetic energy, N-m
t	time, s
t_c	time to center, s
t_r, t_z	times of radial- and vertical-gradient dominance, respectively, s
V	potential energy, N-m
v	wind speed, m/s
v_r, v_z, v_θ	radial, vertical, and tangential velocities, respectively, m/s

v_{rel}	relative wind speed for balloon, m/s
x, y	Cartesian coordinates in horizontal plane
z	altitude, m
z_c	altitude of balloon at center, m
z_t	transition altitude, m
z_ρ	density scale height, m
Γ	circulation of vortex, m^2/s
4ϵ	angular measure of allowable heading error, deg
θ	azimuthal coordinate, radians (positive counterclockwise)
ρ	air density, kg/m^3
ρ_B	mean density of balloon, kg/m^3
ρ_t	transition air density, kg/m^3
ρ_0	ground-level air density

Dot over symbol indicates time derivative.

ANALYSIS

Only constant-volume balloons are considered. The two pressure gradients on the balloon are the radial gradient of the tornado and the vertical general atmospheric gradient, which urge the balloon inward and upward, respectively. It is the radial pressure gradient which supplies the buoyancy force that drives the balloon to the center. In contrast, the ordinary vertical buoyancy force is an impediment because it drives the balloon to higher altitudes where both buoyancies are less. There are two behaviors corresponding to eventual dominance of one or the other gradient. The case of interest is that of dominance of the radial gradient in which the balloon enters the core as shown in figure 1. The closer the tornado is to the launch point, the shorter the time to the

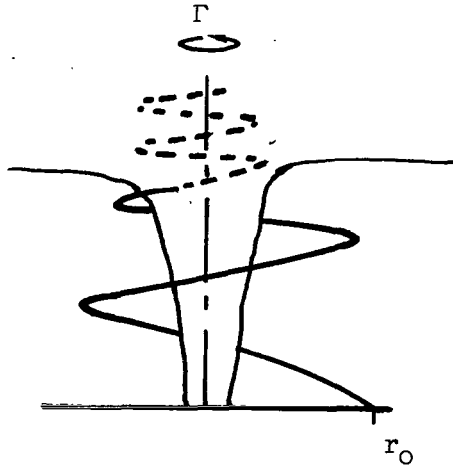


Figure 1.- Balloon trajectory into core.

center, and the lower the altitude at the center. A specified maximum altitude at the center defines a target area, which is a circle centered on the payload. Launches within the target area insure altitudes at the center lower than the specified maximum.

If the balloon is launched too far from the center, it reaches its equilibrium, or float, altitude (where mean balloon density equals air density) before reaching the center. It overshoots and oscillates up and down and in and out until the drag force damps the motion into a steady circulation about the core at equilibrium altitude. There is a dividing case in which the balloon reaches the center at the equilibrium altitude.

Dynamics

The wind speed v in a line-vortex tornado model is

$$v = \frac{\Gamma}{2\pi} \frac{1}{r}$$

where Γ is the circulation of the assumed vertical line vortex. Thus, the only wind is tangential with no inflow or outflow. The vertical atmospheric-density variation used in this model is

$$\rho = \rho_0 \exp\left(-\frac{z}{z_\rho}\right)$$

for air density ρ , ground-level density ρ_0 , altitude z , and density scale height z_ρ . In cylindrical coordinates (fig. 2) the equations of motion are

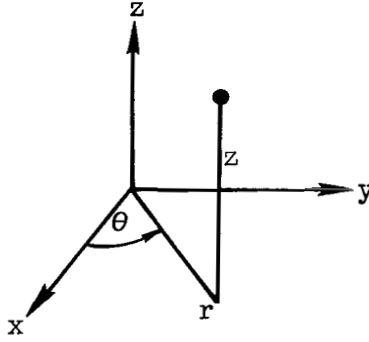


Figure 2.- Coordinate system.

$$M\dot{v}_z = -\frac{\partial p}{\partial z} (\text{Volume}) - F_D \frac{v_z}{v_{\text{rel}}} - Mg \quad (1a)$$

$$M\left(\dot{v}_r - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} (\text{Volume}) - F_D \frac{v_r}{v_{\text{rel}}} \quad (1b)$$

$$M\left(\dot{v}_\theta + \frac{v_r v_\theta}{r}\right) = -F_D \frac{v_\theta}{v_{\text{rel}}} \quad (1c)$$

for mass M , vertical velocity v_z , vertical pressure gradient $\frac{\partial p}{\partial z}$, aerodynamic drag force F_D , wind relative to the balloon v_{rel} , acceleration of gravity g (assumed constant), radial pressure gradient $\frac{\partial p}{\partial r}$, radial velocity v_r , and tangential velocity v_θ .

Specializing for spherical balloons gives

$$M = \rho_B \frac{\pi D^3}{6}$$

for diameter D and mean density ρ_B . The drag force is

$$F_D = C_D A \frac{\rho}{2} v_{\text{rel}}^2$$

for drag coefficient C_D , cross-sectional area A , and relative wind v_{rel} , where

$$A = \frac{\pi D^2}{4}$$

$$v_{\text{rel}}^2 = (v - v_\theta)^2 + v_r^2 + v_z^2$$

The vertical pressure gradient is

$$\frac{\partial p}{\partial z} = -\rho g$$

and the assumed atmosphere is in cyclostrophic balance with the radial gradient

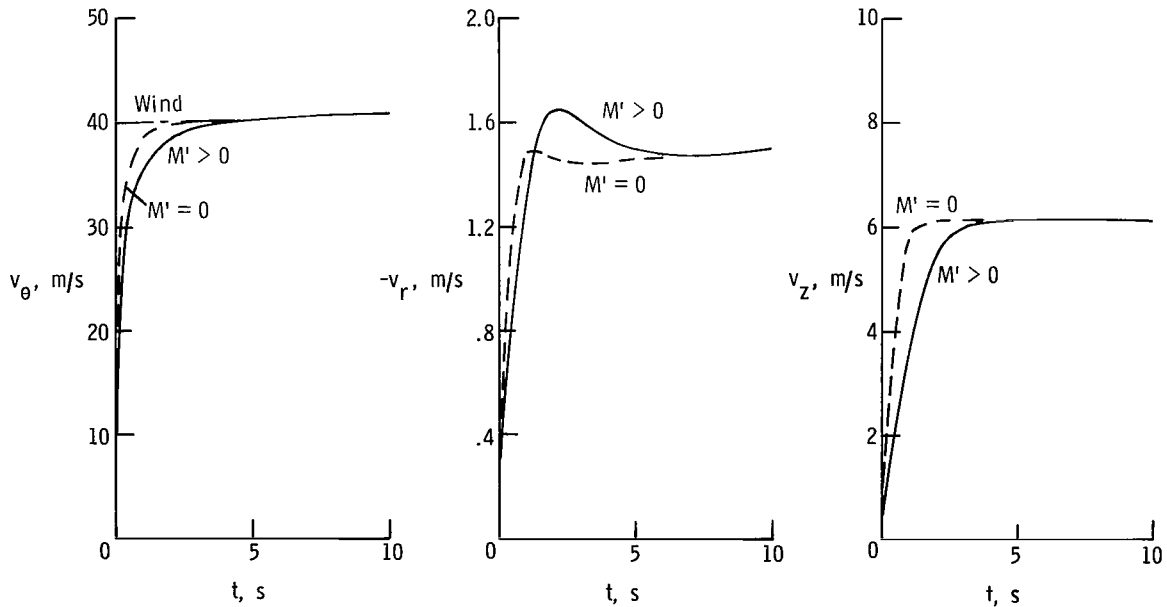
$$\frac{\partial p}{\partial r} = \rho \frac{v^2}{r}$$

while the volume is $\pi D^3/6$. Equations (1) are those used in the approximate analysis.

Effects of Virtual Mass

In balloon motion into a vertical vortex, the effects of virtual mass may be important during the launch and in the final approach to the center when the relative wind may be changing rapidly. The virtual mass of a sphere is half the mass of the displaced fluid. (See ref. 5.) The ordinary theory of virtual mass is for motion into a still fluid; therefore, care must be taken in the application of still-fluid results to motion into a vortex. The derivation of the equations of motion of the virtual mass M' in terms of Lagrangian variables is given in appendix A for vortex flow. In the radial direction an additional inward force appears of the form $-M' \frac{v^2}{r}$. In reference 6 a different approach leads to the additional buoyancy term.

In figure 3 is shown the influence of virtual mass on the motion. The important effects all occur near launch. In figure 3(a) the slower catchup to the wind of the more



(a) Tangential component. (b) Radial component. (c) Vertical component.

Figure 3.- Effect of virtual mass on velocities. $C_D = 0.4$; $r_0 = 700$ m; $D = 2$ m; $\rho_B = 0.5$ kg/m³; $\Gamma = 0.175$ km²/s.

massive ($M' > 0$) balloon is shown. Even with extra mass, the balloon is essentially up to wind speed in about 7 seconds. Earlier, when a large difference between wind and balloon speeds exists, the centrifugal force is too small to effectively impede the radial pressure gradient in driving the balloon to the center. In figure 3(b) the resulting radial velocities are shown. For $M' = 0$ the balloon quickly reaches its quasi-equilibrium state. For $M' > 0$ the balloon is slower at first, but the radial force is larger (appendix A); therefore it has a greater peak velocity and takes longer to reach its quasi-equilibrium state. On balance, there is a slightly nearer initial approach to the center with $M' > 0$. In figure 3(c) the variation in vertical velocity is similar to that in figure 3(a).

Approximations

When \dot{v}_z , \dot{v}_r , and $\dot{v}_\theta + \frac{v_r v_\theta}{r}$ are small (everywhere except near launch or center), the form of the trajectory may be found analytically. The equations of motion become

$$(\rho - \rho_B)D = \frac{3}{4} C_D \rho \frac{v_{rel} v_z}{g} \quad (2a)$$

$$v_\theta = v \quad (2b)$$

$$-\frac{v^2}{r}(\rho - \rho_B)D = \frac{3}{4} C_D \rho v_{rel} v_r \quad (2c)$$

Dividing equation (2c) into equation (2a) yields

$$-r^3 = \frac{1}{g} \left(\frac{\Gamma}{2\pi} \right)^2 \frac{dz}{dr} \quad (3)$$

which integrates to

$$z = \frac{g}{4} \left(\frac{2\pi}{\Gamma} \right)^2 (r_o^4 - r^4)$$

The altitude at the center z_c is

$$z_c = \frac{g}{4} \left(\frac{2\pi}{\Gamma} \right)^2 r_o^4$$

Thus, the altitude at the center is approximately independent of the balloon parameters (mean density ρ_B , drag coefficient C_D , and size D) and depends only on the strength (circulation Γ) of the tornado and the launch radius. To reach the core below the cloud base, the balloon must be released within a fairly well-defined target area because r_o

is such a slowly changing function of z_c . Typical values are 950 meters for z_c and 730 meters for r_o . The time to the center depends on all the parameters, particularly ρ_B , and is not so easily approximated.

Time to Center

The short life of a tornado indicates that a fast approach to the center is necessary to obtain data. All the parameters (ρ_B, C_D, D, Γ) may be important. Along the trajectory the vertical buoyancy dominates at first but eventually the radial buoyancy dominates. It is physically reasonable to regard the point at which the quantity $-dz/dr$ becomes unity as a transition for dominance. At this point (r_t, z_t) equation (3) indicates that

$$r_t = \sqrt[3]{\frac{1}{g} \left(\frac{\Gamma}{2\pi} \right)^2}$$

and

$$z_t = \frac{g}{4} \left(\frac{2\pi}{\Gamma} \right)^2 (r_o^4 - r_t^4)$$

Before the balloon reaches (r_t, z_t) , the approximate relative wind is

$$v_{rel} \approx v_z$$

and from equation (2a)

$$(\rho - \rho_B)D = \frac{3}{4} C_D \rho \frac{v_z^2}{g}$$

Solving for $v_z = dz/dt$ and integrating yields

$$t_z = \sqrt{\frac{3C_D}{gD}} z_\rho \log \left(\frac{\sqrt{\rho_o} + \sqrt{\rho_o - \rho_B}}{\sqrt{\rho_t} + \sqrt{\rho_t - \rho_B}} \right)$$

where

$$\rho_t = \rho_o \exp \left(-\frac{z_t}{z_\rho} \right)$$

Beyond (r_t, z_t) the approximate relative wind is

$$v_{rel} \approx v_r$$

and from equation (2c)

$$\left(\frac{\Gamma}{2\pi}\right)^2 \frac{1}{r^3} (\rho - \rho_B) D = \frac{3}{4} C_D \rho v_r^2$$

Because the altitude does not change much when radial buoyancy dominates, the density factor is approximately constant; therefore, solving for v_r and integrating yields

$$t_r = \frac{2\pi}{5\Gamma} \sqrt{\frac{3C_D}{D}} \sqrt{\frac{\rho_t}{\rho_t - \rho_B}} r_t^{5/2}$$

The time to the center t_c is the sum

$$t_c = t_r + t_z$$

Results based on these formulas will be given subsequently. The differences found between times and trajectories calculated by analytic approximation and those computed by numerical integration including virtual mass (appendix B) have been slight – several percent.

Note that since $z_t \approx z_c$, the formulas indicate the impossibility of reaching the core for float altitudes less than altitude at the center z_c . When the balloon reaches float altitude, $\rho = \rho_B$ and vertical and horizontal buoyancy both vanish. The approximate trajectory analysis seems to produce core entry for every initial radius. In fact, as mentioned already, there is a starting radius beyond which the balloon cannot reach the core. This radius is the one for which the altitude at the center z_c equals the float altitude. However, the regime of interest is that for which the float altitude far exceeds the cloud-base height.

EXPERIMENTAL PROCEDURE

Launch Phase

The basic experimental plan is indicated in figure 4. The airplane comes down the track of the tornado while the crew obtains the tornado bearing and estimates its strength and speed. The airplane passes on the downwind side in order to have the safer high-air-speed direction away from the core. Keeping a safe distance (several kilometers), it drops a pair of instrumented self-inflating balloons at a distance S ahead of the core and at a distance r_0 on each side of the estimated track. When the tornado enters either launch circle, the balloon quickly inflates to rise up and in. (It very quickly

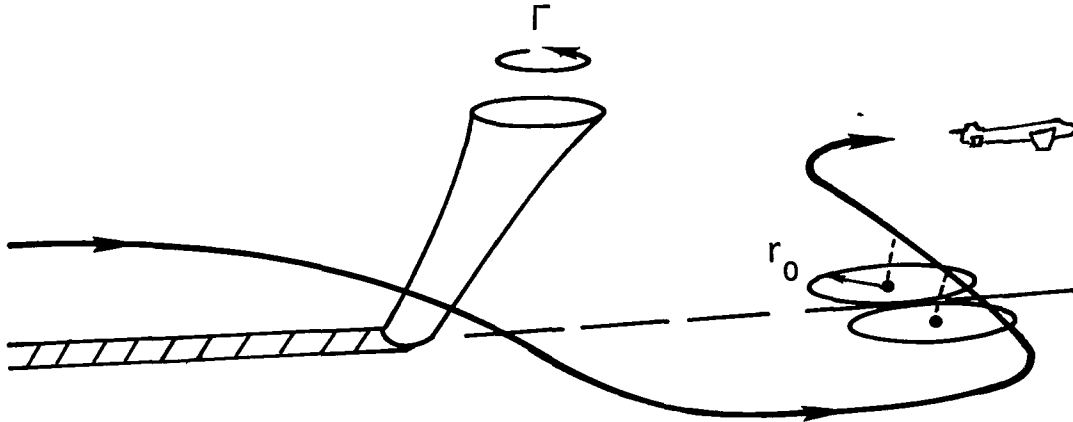
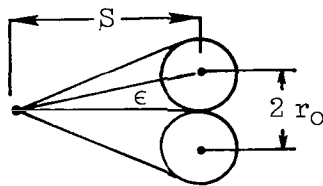


Figure 4.- Schematic drawing of approach to tornado and of instrument launch.

gains and maintains the fast rotational speed of the tornado winds, as shown in figure 3.) If one dared fly as close as r_0 to the center, the balloons could be released in midair. Ground-dropping ahead permits the airplane to stay far from the center. The lead distance S in units of launch radius r_0 , and the corresponding error angles 4ϵ are indicated in table I.

TABLE I.- LEAD DISTANCES AND ANGULAR ERRORS



S/r_0	4ϵ , deg
3	74
4	56
5	45
6	38

For example, a launch radius of 700 meters permits a drop 4200 meters (2.6 miles) ahead with allowable total error of about 38° in tornado heading. More than two balloons will provide proportionately larger margin for error or greater approach distances.

The payload must survive the airdrop; therefore, its fall will be checked by a small parachute. On the ground, it will sense the approach of the tornado by, for example, monitoring the static-pressure drop. At the minimum pressure or at a pressure corresponding to the highest wind the balloon can withstand, the balloon suddenly inflates and rises with its instruments. These instruments might consist of transponders modulated by temperature, humidity, electric field, ionization, or other physical properties. Either the launch plane itself, another tracking plane, or ground radar can record the trajectory

as the balloon is drawn up and into the core. Additional heavy instrumentation can be dropped on the track to await the close passage of the tornado core. Also, low-float-altitude balloons might be air launched on the approach.

Pursuit Phase

If a scale larger than the neighborhood of the tornado is considered, the remaining fundamental problem appears: How can the vicinity of the tornado be reached in a small fraction of its lifetime? To state the solution is easy enough, but its construction involves simultaneous consideration of the statistical distribution of tornado parameters. Tornadoes have a wide range of parameter values, a sampling of which is indicated in table II (from ref. 7), in which values have been converted into SI units. In addition, there are a geographic distribution and a calendar distribution.

TABLE II.- TORNADO PARAMETERS

	Typical range	Weighted average
Visual cloud-base height	0.3 to 1.5 km	0.9 km
Lifetime	15 to 60 min	20 min
$\Gamma/2\pi$	0.01 to 0.05 km ² /s	0.028 km ² /s

Consider an airborne research team consisting of a launch plane and a trained crew (with possibly another tracking plane and crew). Obviously, the research team should be near the right place at the right time to maximize the probability of sighting long-lived tornadoes in daylight. Certain areas and seasons are thus indicated. The weather bureau issues tornado watches (on a time scale of hours), which typically are for areas whose dimensions correspond to about an hour's flying time. Thus, it is possible for a research team to be within a fraction of an hour's flight time of a tornado. The characteristic hook-shaped ground-radar pattern identifies possible fresh tornadoes. Probably, more than one attempt will be made before the research team puts down its payload on the track of a satisfactory tornado.

The stochastic nature of the quarry makes a particular chase uncertain, but, on the other hand, eventual success of the hunt is certain. A crude estimate of interception possibilities can be made on the assumption of a 500-km/hr airplane operating from a fixed base in search of tornadoes of 20-minute lifetime. If the base is located in the central United States, where the maximum reported frequency is one per year per 80 kilometers square (ref. 8), the indicated effective radius of 170 km implies $\approx 10^1$ interceptions per year. It may be noted that waterspouts have been successfully approached and studied scientifically from aircraft. (See ref. 9.) Waterspouts, however, are more common than tornadoes and have weaker circulations.

RESULTS AND DISCUSSION

It has been shown that the height at the core depends strongly on the circulation Γ . It is thus possible to obtain, for typical ranges of cloud-base height and tornado strength, an envelope defining values of launch (target) radius to be expected in natural tornadoes. Such an envelope is indicated in figure 5. The range is from 300 meters for a weak low-cloud-base tornado to about a kilometer for a strong, high-cloud-base tornado. The weighted-average tornado, indicated with a dot, has a radius of 730 meters and a cloud-base height of 950 meters. The two instrument packages of figure 4 can thus, on the average, be dropped several kilometers ahead of the tornado. Even then, only large deviations in its path can cause the tornado to miss both target areas. (See section entitled "Launch Phase.")

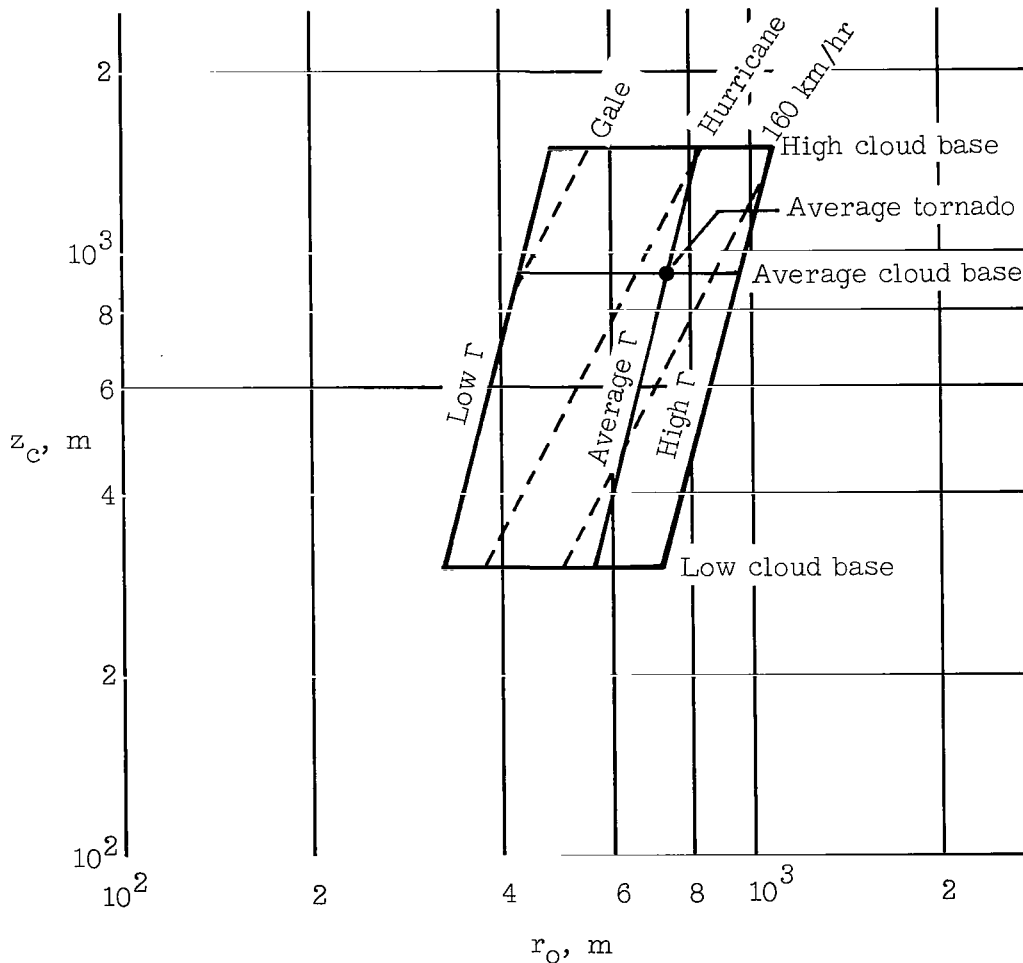
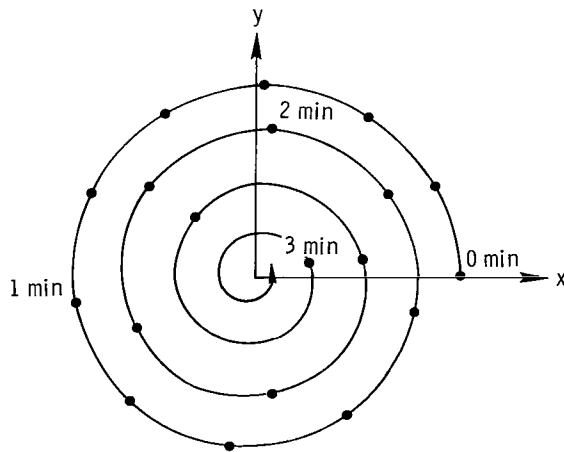


Figure 5.- Launch radius as defined by cloud-base height and tornado circulation.

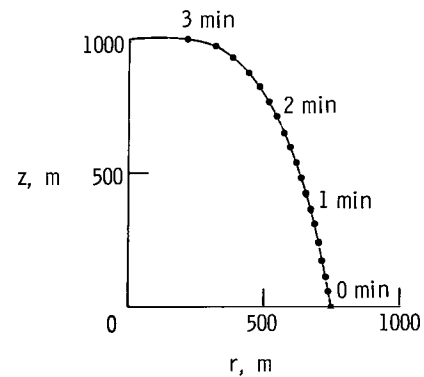
An important consideration is the ability of the balloon to withstand the winds while it accelerates to wind speed after its inflation. Lines of constant wind velocity are drawn within the envelope in figure 5. To completely cover the range of launch conditions, the balloon must be able to survive winds of more than 160 km/hr. If average or weaker than average tornadoes are considered, the requirement reduces to withstanding slightly higher than hurricane winds. A balloon that can withstand only gale winds will survive only in a small part of the envelope. To be viable in most of the envelope, a tornado-probing balloon must hold together in winds somewhat greater than hurricane force. However, in 1 second (fig. 3(a)) high relative winds become low.

After inflation the balloon quickly accelerates to the wind speed and moves in the much weaker relative winds induced by buoyancy gradients. For simplicity, spherical shapes have been considered, but streamline forms with lower drag may ease the launch wind problem while speeding up the entry time. If a maximum size of balloon for a given wind is postulated, it is evident that small balloons may be arranged in tandem to lift a given weight. The penalty is slight since the time to the center will increase as the $1/6$ power of the number used. For n balloons of diameter d and total volume $\pi D^3/6$, $\frac{d}{D} = n^{-1/3}$. The new drag coefficient based on $\frac{\pi D^2}{4}$ is $C_D' \propto n \frac{d^2}{D^2}$ so that $t_c \propto \sqrt{C_D'} \propto n^{1/6}$.

Although it is plainly the safest course to fly far from tornadoes, the possibility of an air launch should not be ignored. In figure 6 the plan view and r - z history of a balloon ground launched at 750 meters is shown. If an airplane could fly as close as



(a) Plan view.



(b) View of moving plane of vortex and balloon.

Figure 6.- Typical balloon trajectory. $C_D = 0.4$; $D = 2$ m; $\rho_B = 0.5$ kg/m³; $\Gamma = 0.175$ km²/s; $\Gamma_D = 750$ m.

600 meters at 400 meters altitude, an air-launched balloon would enter the core more quickly and at lower altitude than the ground-launched balloon. Air launch at any point on the r-z trace is exactly equivalent to ground launch.

The effects of buoyancy on times to the center are shown in figure 7 for low-, average-, and high-circulation tornadoes. Two cases of buoyancy are shown. In one, a balloon containing hydrogen is considered (density 0.0845 kg/m^3); in the other, a balloon weighted down to a mean density of 0.5 kg/m^3 is considered. (Ground-level air density is 1.225 kg/m^3 .) The idealized pure-hydrogen balloon gives a measure of the fastest possible entry to the core. It is interesting that the loaded balloon is slower but not very much slower; the principal effect on entry time is the tornado strength.

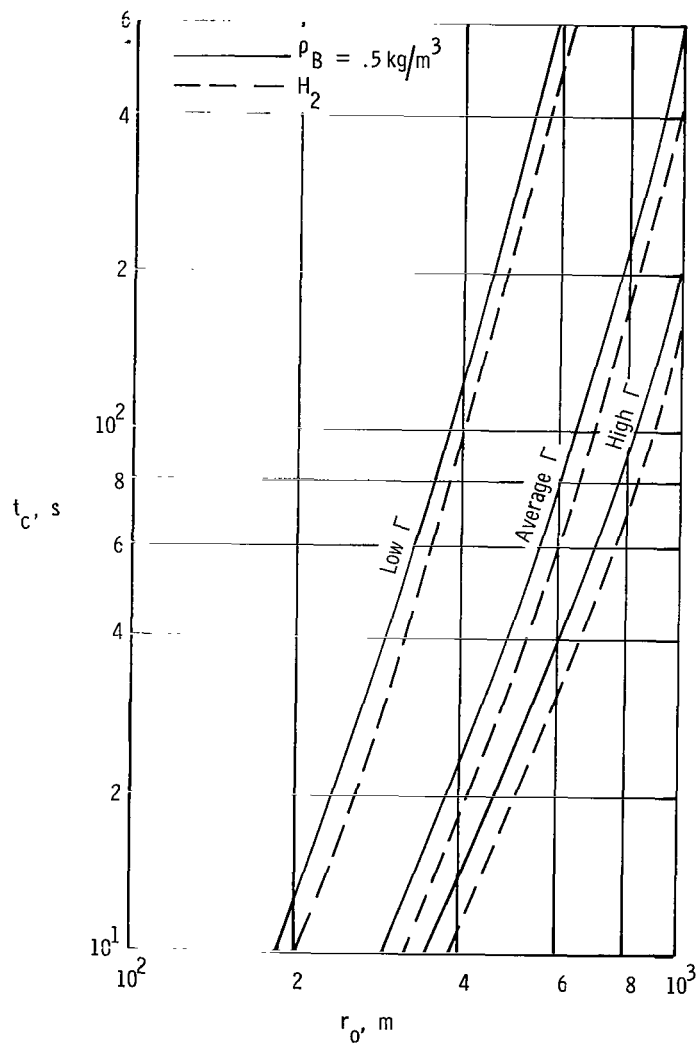


Figure 7.- Times to center as a function of launch radius. $C_D = 0.4$; $D = 2 \text{ m}$.

The times to center are proportional to $\sqrt{C_D/D}$. To obtain a feel for the numbers, a balloon of 2-meter diameter is considered in figure 7. Launched at a radius of 750 meters, a hydrogen balloon reaches the center in 2 minutes 20 seconds. With 1700 grams of load, the balloon hits the center in about 3 minutes (a drag coefficient of 0.4 is assumed). With a smaller load and a streamline shape, the balloon may be reduced in size and still maintain comparable times to the center. Thus, an entry time of several minutes seems quite feasible for the smaller balloons, which better withstand launch winds. Several minutes are a sufficiently small fraction of an average tornado lifetime that a condition for high probability of successful entry is satisfied.

Within the envelope shown in figure 5 the appearance of every point (tornado) has been tacitly regarded as equally probable, but there may be correlations between cloud-base height and circulation just as there is a correlation between funnel proportions and circulation – thicker, squatter tornadoes have stronger circulations (ref. 7).

CONCLUDING REMARKS

It has been shown how, without passing closer to the core than several kilometers, airplanes may be used to introduce instrumented balloons into the cores of tornadoes at or below the cloud base. The times required are short enough that a large fraction of tornadoes are suitable for the technique. The automatic centering action of the radial pressure gradient in driving the balloon to the core requires only a proper launch radius to be effective.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., September 22, 1971.

APPENDIX A

MOTION OF BALLOON WITHOUT INERTIAL MASS MOVING INTO VERTICAL VORTEX

Ordinarily, the virtual mass M' is simply added to the inertial mass M . In a still fluid the kinetic energy T of a massless ($M = 0$) sphere is given by

$$2T = M'(\dot{v}_z^2 + \dot{v}_r^2 + \dot{v}_\theta^2)$$

where $M' = \frac{\text{Mass of displaced fluid}}{2}$. For a vertical vortex,

$$2T = M'[\dot{v}_z^2 + \dot{v}_r^2 + (\dot{v}_\theta - v)^2] \quad (\text{A1})$$

Lagrange's equations for the quasi-particle of equation (A1) are

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} \right) = Q_i$$

The Lagrangian function L equals T because $V = 0$ where $L = T - V$. By taking r, θ, z as coordinates q_i ,

$$2L = 2T = M'[\dot{z}^2 + \dot{r}^2 + (r\dot{\theta} - v)^2]$$

and

$$M'\dot{v}_z = F_z \quad (\text{A2a})$$

$$M' \left(\dot{v}_r - \frac{v\dot{\theta}}{r} \right) = F_r + M' \left[-\frac{v\dot{v}_\theta}{r} - \frac{dv}{dr}(v_\theta - v) \right] \quad (\text{A2b})$$

$$M'(r\dot{v}_\theta + v_\theta \dot{v}_r) = rF_\theta - M'(-r\dot{v} - v\dot{v}_r) \quad (\text{A2c})$$

where F_z, F_r, F_θ are the applied forces in the z -, r -, θ -directions, respectively. The first member in parentheses on the right-hand side of equation (A2c) corresponds to a term which appears in the wind-shear case treated in reference 10, but cancels here for the irrotational flow outside the line vortex. The Lagrangian for the wind-shear

APPENDIX A – Concluded

case produces the apparent mass term of reference 10. In the assumed model $2\pi vr = \Gamma$ so $\frac{dv}{dr} = -\frac{v}{r}$ and $\dot{v} = \frac{dv}{dr} \cdot \frac{dr}{dt} = -\frac{vv_r}{r}$. Hence, finally,

$$M'\dot{v}_z = F_z \tag{A3a}$$

$$M'\left(\dot{v}_r - \frac{v_\theta^2}{r}\right) = F_r - M'\frac{v^2}{r} \tag{A3b}$$

$$M'\left(\dot{v}_\theta + \frac{v_r v_\theta}{r}\right) = F_\theta \tag{A3c}$$

From equations (A3) it can be seen that the virtual mass is inertial except in the radial direction, where a new force field $-M'\frac{v^2}{r}$ appears. These considerations are an application of still fluid results to vortex flow; therefore the approximation is best when $D \ll r$ and the velocity perturbation field of the balloon extends significantly only a small fraction of the radius to the balloon. Physically, the term $-M'\frac{v^2}{r}$ corresponds to the increased velocity on the vortex side of the sphere in the velocity gradient of the vortex. The increased velocity causes a lower pressure and an additional buoyancy force toward the center.

APPENDIX B

MOTION EQUATIONS INCLUDING VIRTUAL-MASS EFFECTS

The quantities entering into the FORTRAN IV equations for VHDOT, VRDOT, and VTHDOT (coordinate accelerations for vertical, radial, and tangential directions) are

REAL MV, M, M1

G acceleration of gravity, 9.80665

VH vertical velocity

VR radial velocity

VTH tangential velocity

V vortex wind speed

RHO air density

$VRELTH = VTH - V$ tangential relative wind

M balloon mass

$W = M * G$ balloon weight

AREA balloon cross-sectional area

VOL balloon volume

MV balloon virtual mass

$M1 = 1. / (M + MV)$

R1 inverse of radial position

$GRADPH = -RHO * G$ vertical pressure gradient

APPENDIX B – Concluded

$$\text{GRADPR} = \text{RHO} * \text{V}^{**2} * \text{R1} \quad \text{radial pressure gradient}$$

$$\text{VREL} = \text{SQRT}(\text{VH}^{**2} + \text{VR}^{**2} + \text{VRELTH}^{**2}) \quad \text{relative wind speed}$$

$$\text{VREL1} = 1./\text{VREL}$$

$$\text{CD} \quad \text{drag coefficient}$$

$$\text{FD} = \text{CD} * \text{AREA} * \text{VREL} * \text{RHO} * .5 \quad \text{Drag/Relative wind speed}$$

$$\text{VDOT} = -\text{V} * \text{R1} * \text{VR} \quad \text{rate of wind change along radius}$$

The equations of motion themselves are

$$\text{VHDOT} = -(\text{GRADPH} * \text{VOL} + \text{W} + \text{FD} * \text{VH}) * \text{M1} \quad (\text{B1a})$$

$$\text{VRDOT} = \text{VTH}^{**2} * \text{R1} - (\text{GRADPR} * \text{VOL} + \text{FD} * \text{VR} + \text{MV} * \text{V}^{**2} * \text{R1}) * \text{M1} \quad (\text{B1b})$$

$$\text{VTHDOT} = -\text{VR} * \text{VTH} * \text{R1} - \text{FD} * \text{VRELTH} * \text{M1} \quad (\text{B1c})$$

$$\text{HDOT} = \text{VH} \quad (\text{B1d})$$

$$\text{RDOT} = \text{VR} \quad (\text{B1e})$$

$$\text{THDOT} = \text{VTH} * \text{R1} \quad (\text{B1f})$$

Equations (B1) were integrated with a standard Langley Research Center fourth-order predictor-corrector technique which gets starting values by fourth-order Runge-Kutta integration.

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